CONVOLUTION CODING

(23)

Convolution Encoding

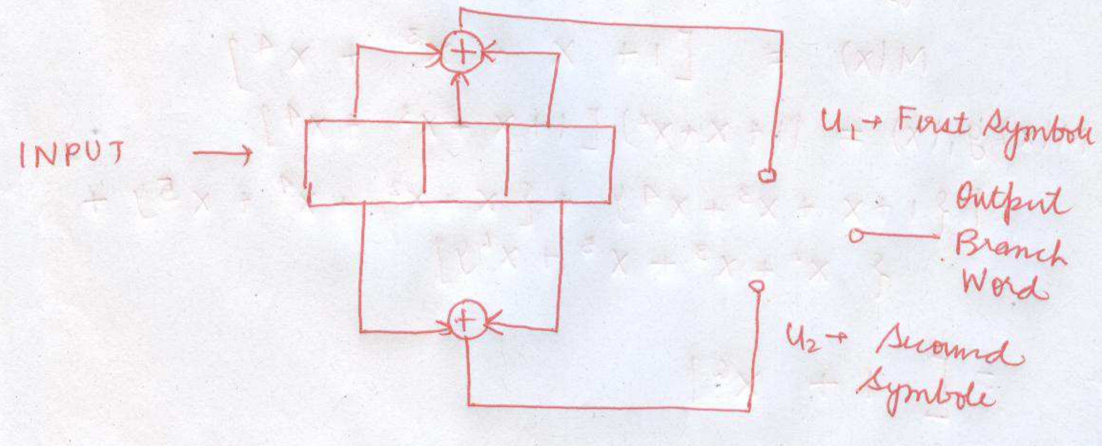
K = Number of Memory element (FF)

n = number of input bits

K = number of output bit for the input sequence

$\eta = \frac{K}{n}$ = Code Rate ~ 0.5 for the high values of n

A conventional encoder is given as



Generator polynomial for the above connection is written as

$g_1(x) = [1 + x + x^2]$

$g_2(x) = [1 + x^2]$

Message $M = [m_0 \; m_1 \; \dots \; m_{N-1}]$ is written in the form of polynomial as

$M(x) = [m_0 x^{(N-1)} + m_0 x^{(N-1)} + \cdots + m_{(N-1)}]$ (24)

Let a message stream is given as

$M(x) = [1 \; 1 \; 0 \; 1 \; 1]$

We have to draw encoder state diagram, tree representation of the encoder and encoder trellis for the encoder present in the last page

Now calculate the output with the help of generator polynomial

$g_1(x) = [1 + x + x^2]$

$g_2(x) = [1 + x^2]$

$M(x) = [1 + x + x^3 + x^4]$

$M(x)\, g_1(x) = (1 + x + x^2)[1 + x + x^3 + x^4]$

$= [\{1 + x + x^3 + x^4\} + \{x + x^2 + x^4 + x^5\} + \{x^2 + x^3 + x^5 + x^6\}]$

$= [1 + x^6]$

$M(x)\, g_2(x) = (1 + x^2)(1 + x + x^3 + x^4)$

$= [\{1 + x + x^3 + x^4\} + \{x^2 + x^3 + x^5 + x^6\}]$

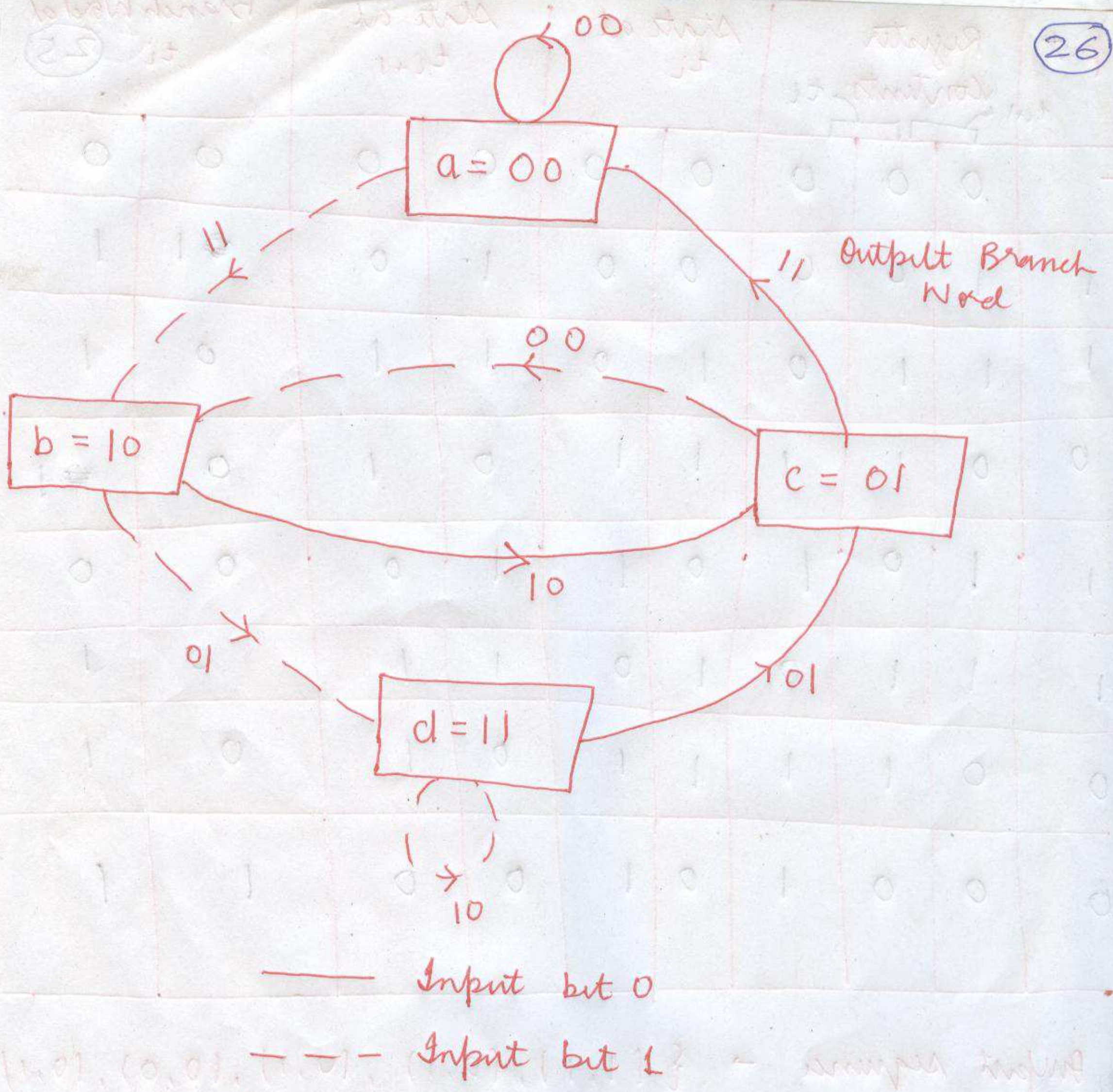
$= [1 + x + x^2 + x^4 + x^5 + x^6]$

Now output can be represented as

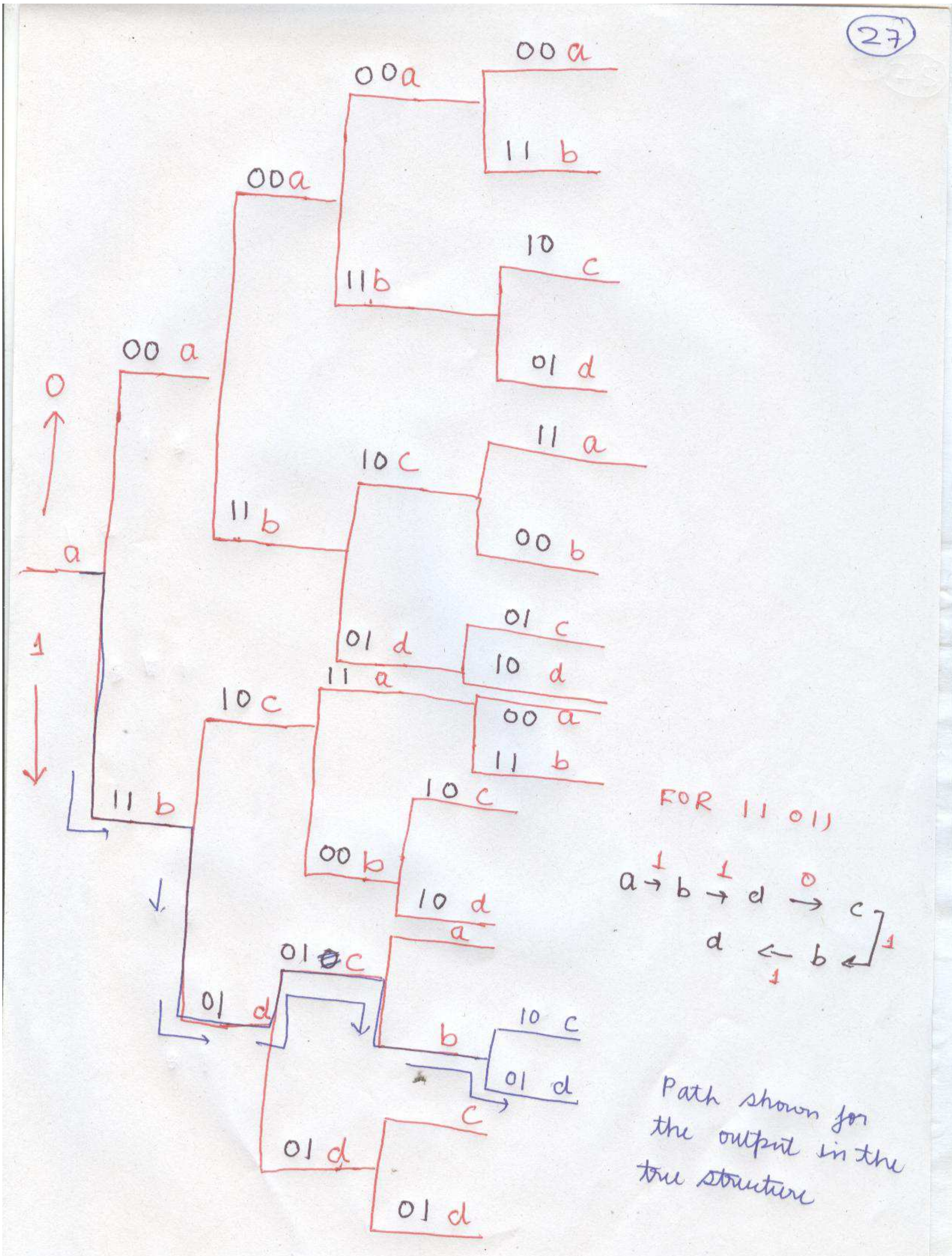
$U(x) = [(1,1), (0,1), (0,1), (0,0), (0,1), (0,1), (1,1)]$

Input	Register contents (t_{i+1})		(t_i)	State at t_i		State at t_{i+1}		Branch Word at t_i	
—	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	1	0	1	1
1	1	1	0	1	0	1	1	0	1
0	0	1	1	1	1	0	1	0	1
1	1	0	1	0	1	1	0	0	0
1	1	1	0	1	0	1	1	0	1
0	0	1	1	1	1	0	1	0	1
0	0	0	1	0	1	0	0	1	1

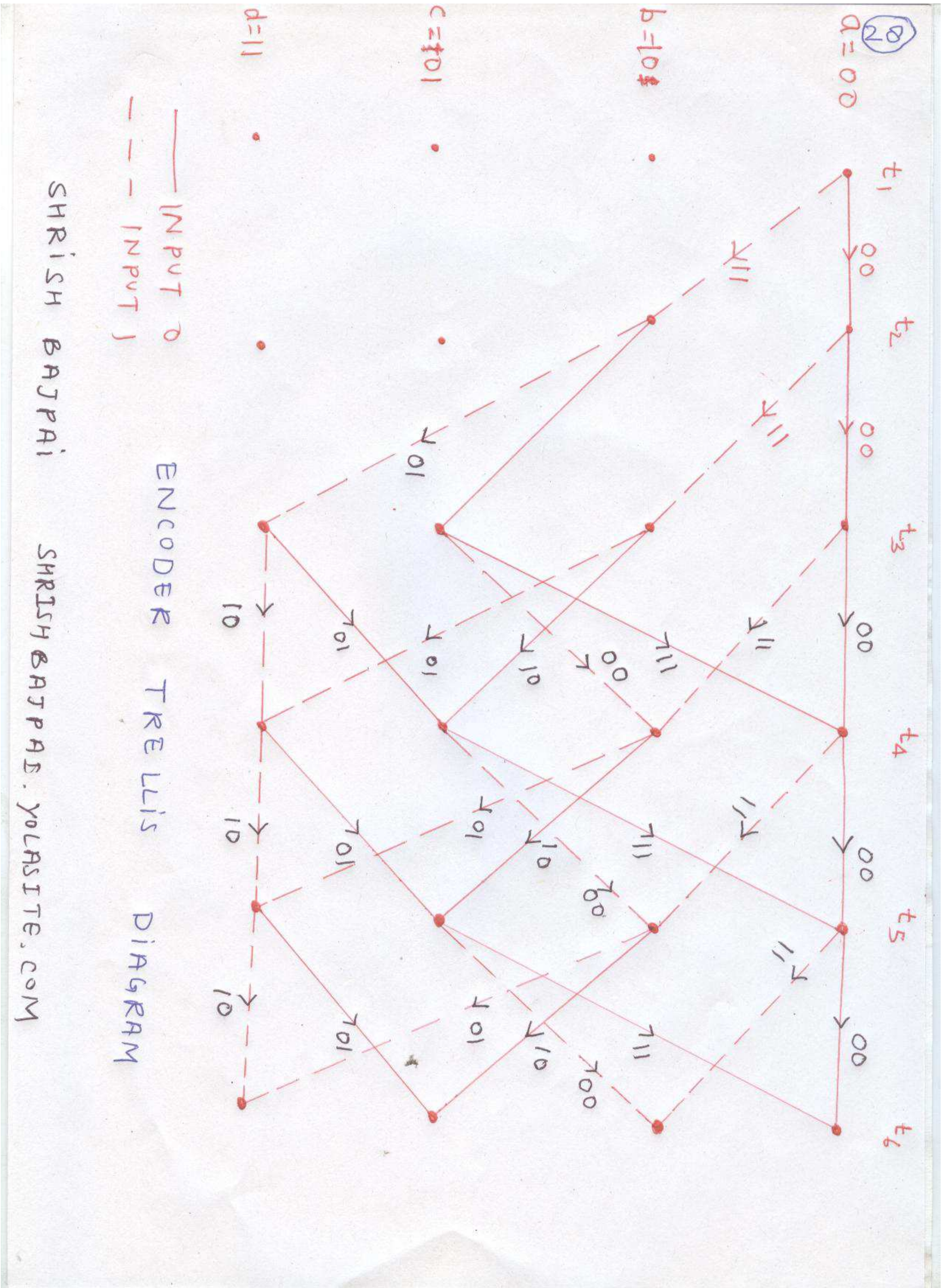
Output sequence → {(1,1), (0,1), (0,1), (0,0), (0,1), (0,1), (1,1)}



Encoder State Diagram



TREE REPRESENTATION OF ENCODER


28
a = 00
b = 10
c = 01
d = 11
t1
t2
t3
t4
t5
t6
INPUT 0
INPUT 1
ENCODER TRELLIS DIAGRAM
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Decoding of Convolution Code

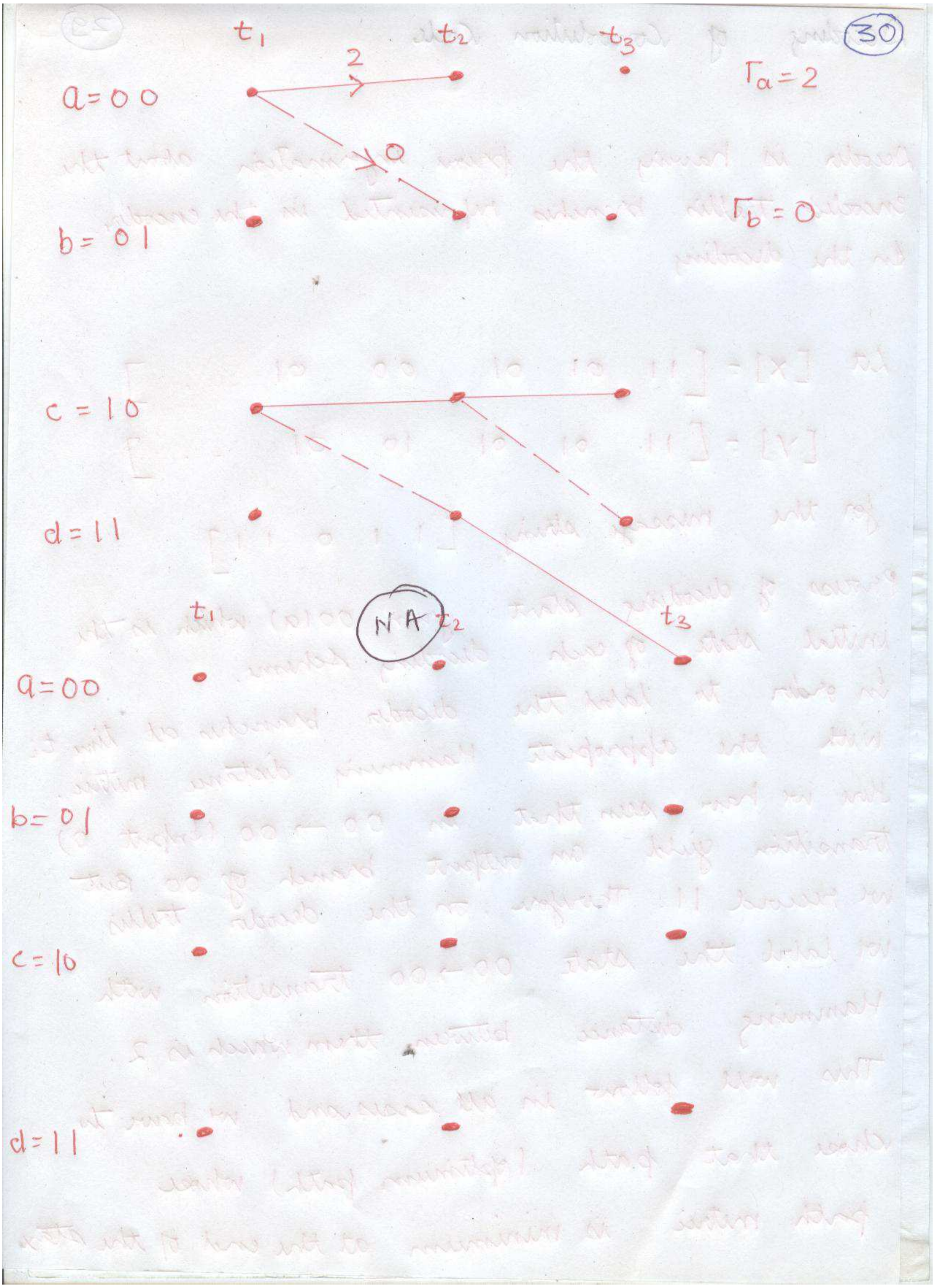
(29)

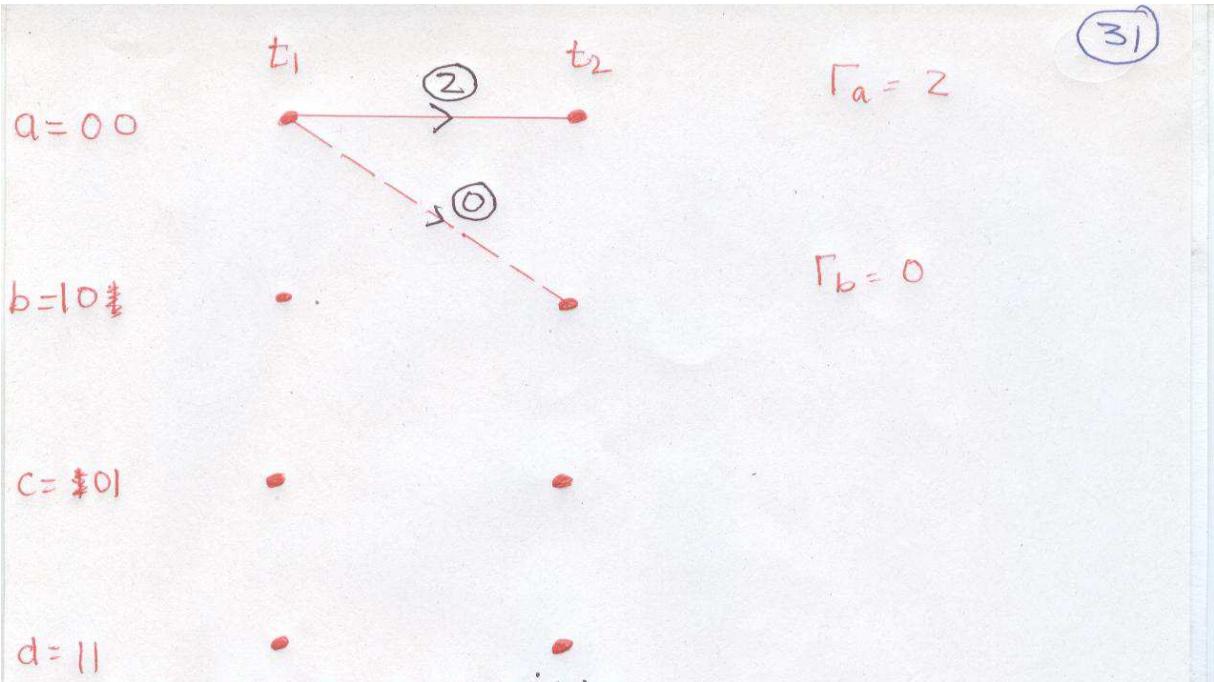
Decoder is having the prior information about the encoded trellis branches represented in the encoder. In the decoding

Let [X] = [11 01 01 00 01 ...]
[Y] = [11 01 01 10 01 ..]

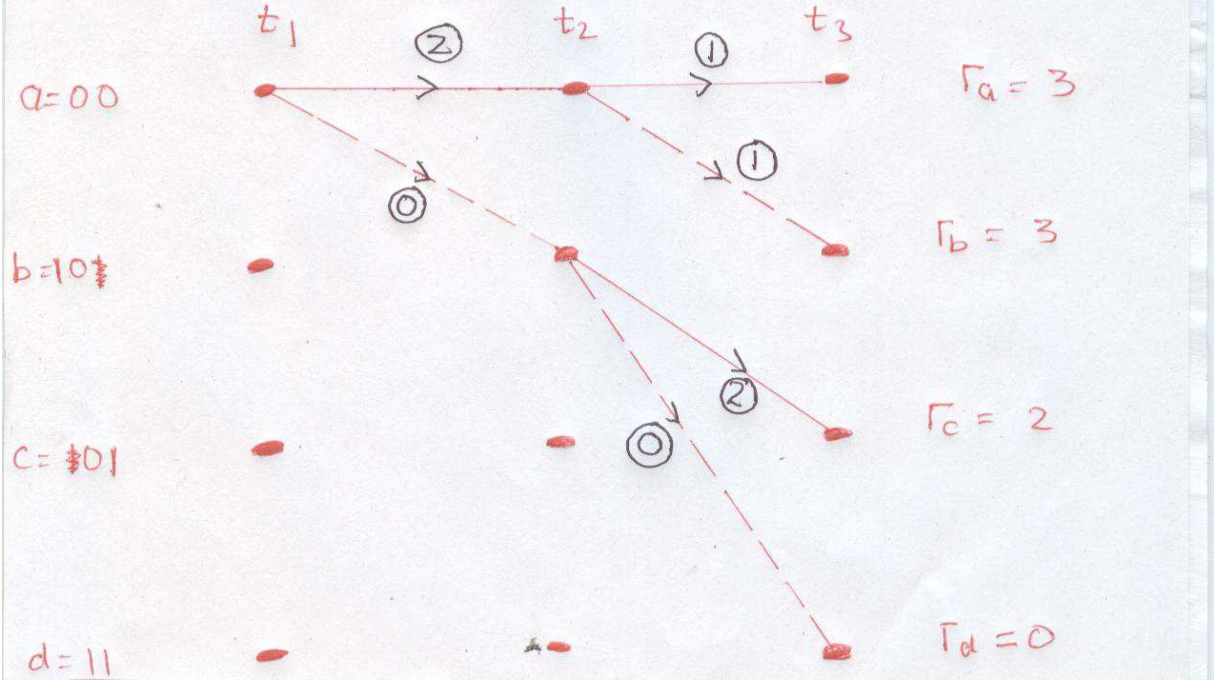
for the message string [1 1 0 1 1]

Process of decoding start from 00(a) which is the initial state of each decoding scheme.
In order to label the decoder branches at time t_i with the appropriate Hamming distance metric.
Here we have seen that in 00 → 00 (Input 0) transition yield an output branch of 00. But we record 11. Therefore, on the decoder trellis we label the state 00 → 00 transition with Hamming distance between them which is 2.
This will follow in all cases, and we have to chose that path (optimum path) whose path metric is minimum at the end of the stage


30
t1
t2
t3
2
0
a=00
b=01
c=10
d=11
Γa=2
Γb=0
NA
t1
t2
t3
a=00
b=01
c=10
d=11



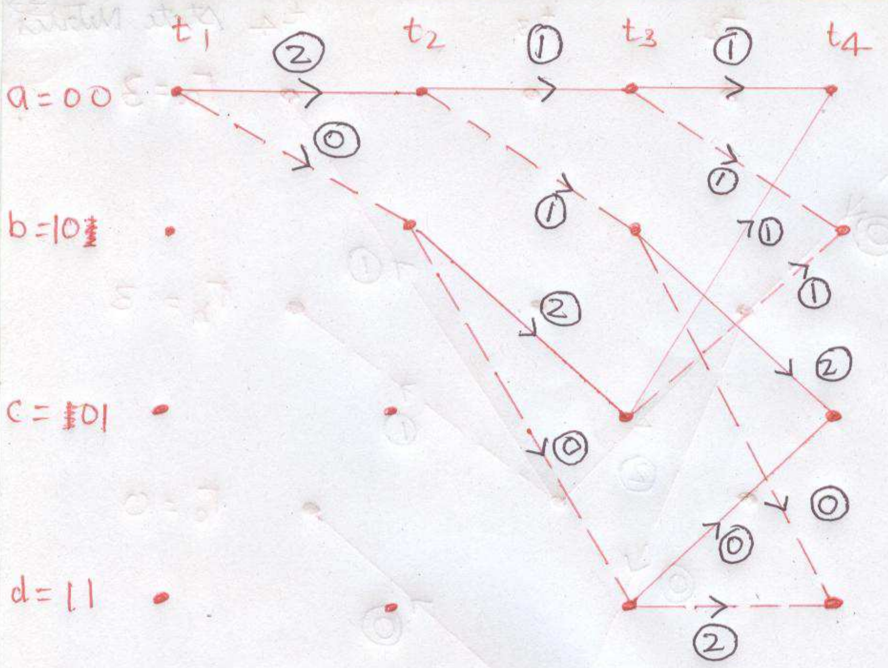
FOR FIRST TWO DIGIT



	H·D
00 – 01	– 1
01 – 01	– 0
10 – 01	– 2
11 – 01	– 1

FOR SECOUND TWO DIGIT

32



FOR THE THIRD TWO DIGIT

		H·D
00	01	1
01	01	0
10	01	2
11	01	1

State Metrics

$\Gamma_a = 4$

$\Gamma_b = 4$

$\Gamma_c = 5$

$\Gamma_d = 3$

$\Gamma_e = 3$

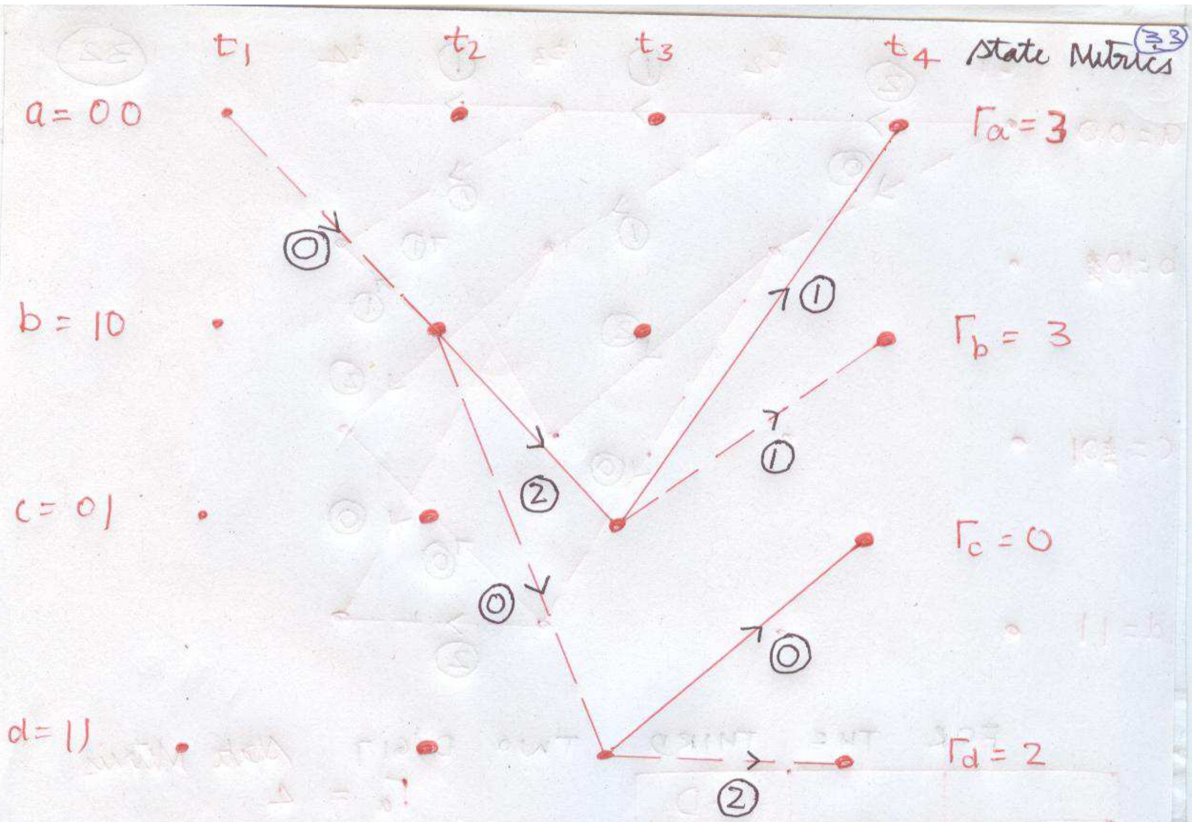
$\Gamma_f = 3$

$\Gamma_g = 0$

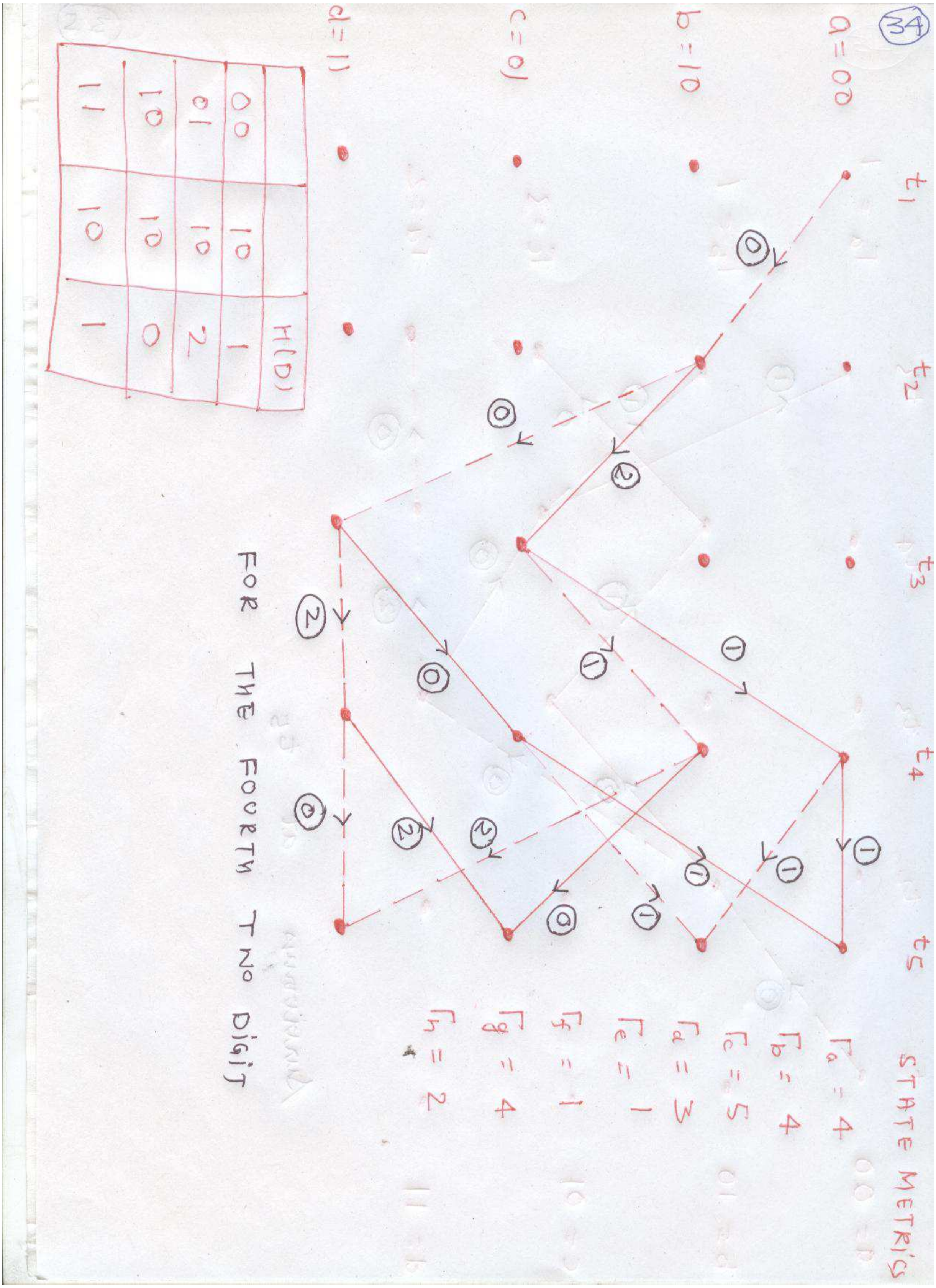
$\Gamma_h = 2$

Γ_d is not selected because similar type path exist Γ_h.

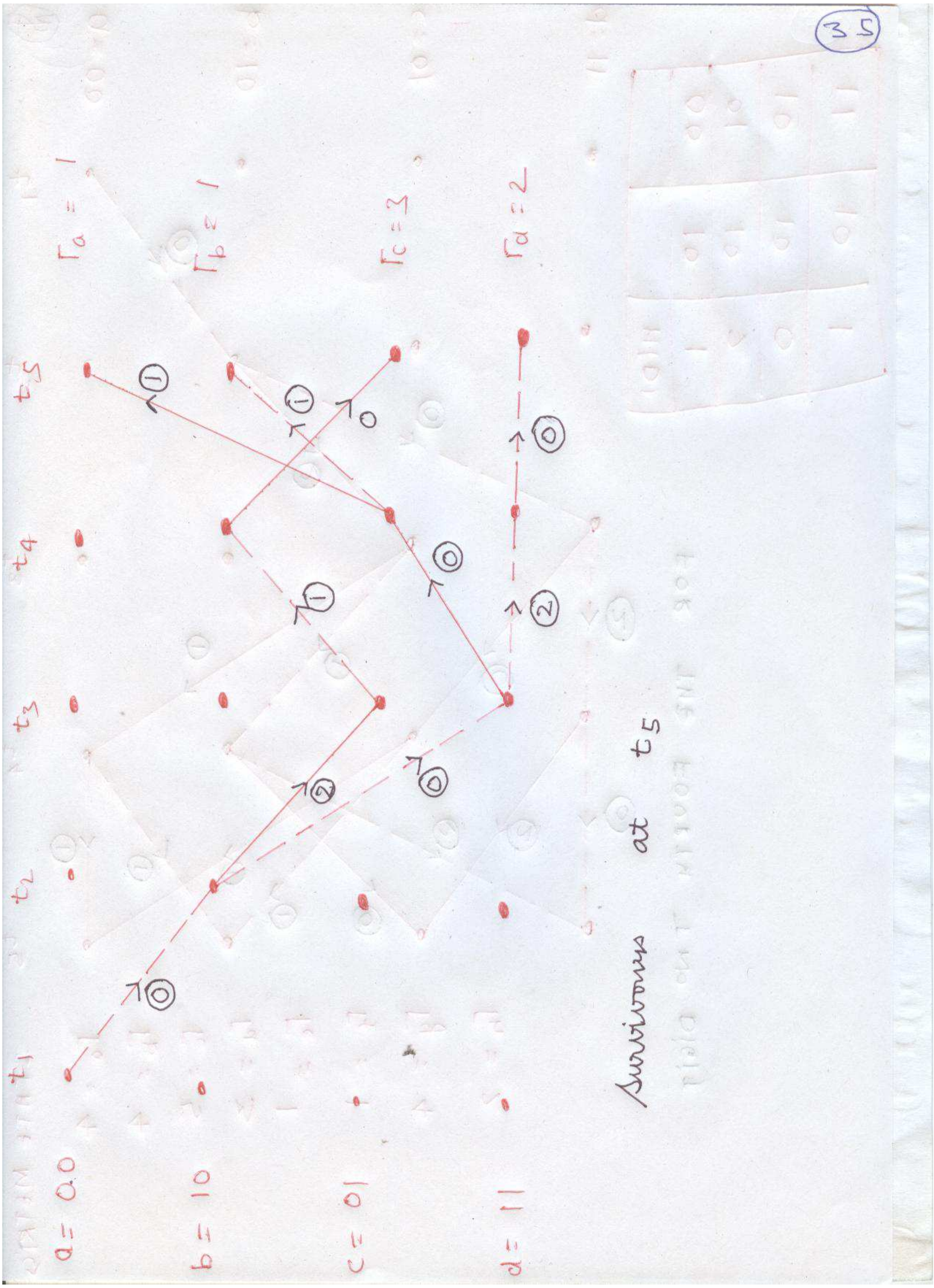
Selecting Γ_e, Γ_f, Γ_g, Γ_h from the above survivor path through metric comparison

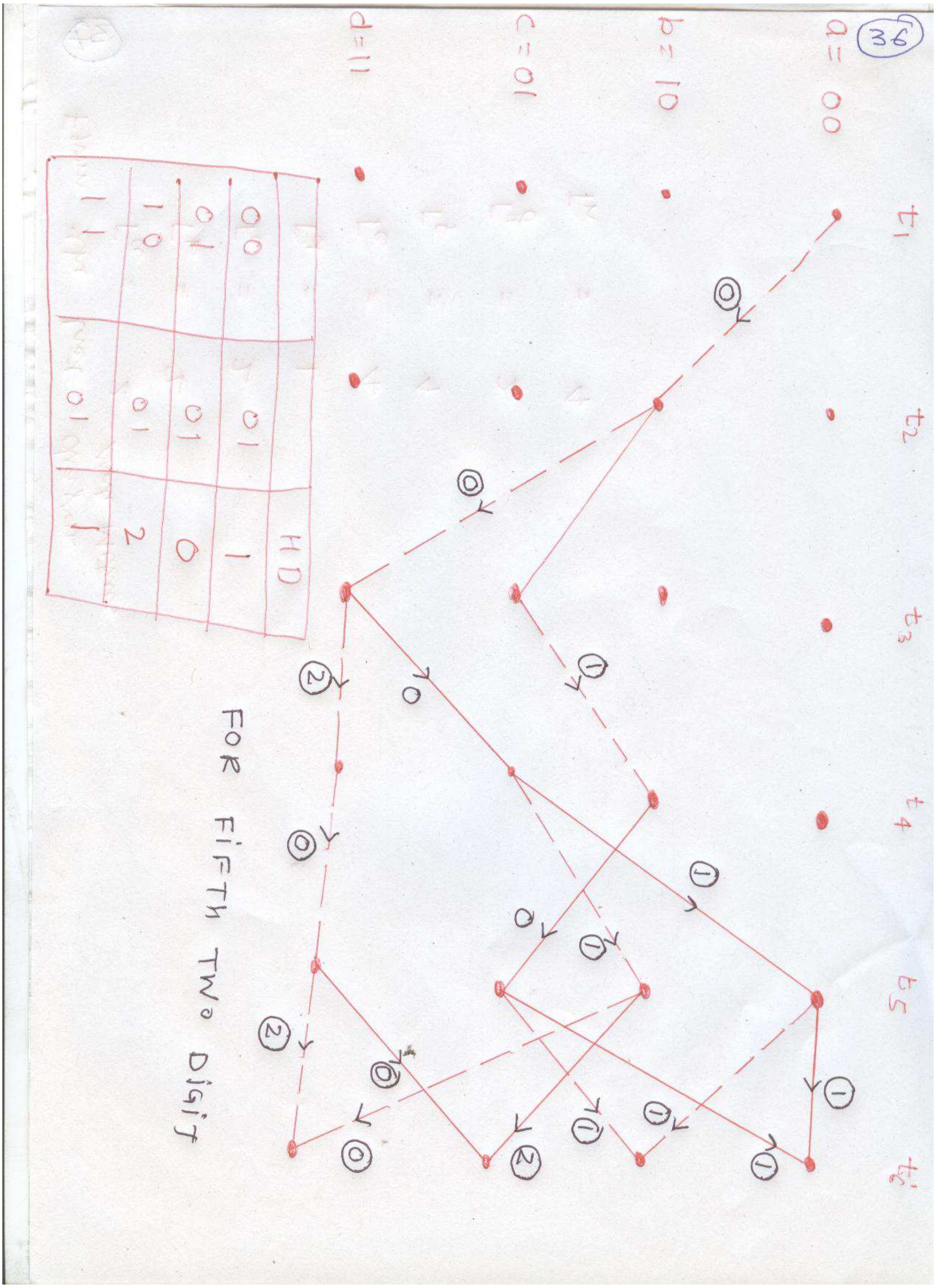


Survivors at t_4 [After third two digits entered in the encoder]



		H(D)
00	10	1
01	10	2
10	10	0
11	10	1


35
a = 00
b = 10
c = 01
d = 11
t1
t2
t3
t4
t5
Γa = 1
Γb = 1
Γc = 3
Γd = 2
Survivors at t5


36
a = 00
b = 10
c = 01
d = 11
t1
t2
t3
t4
t5
t6
FOR FIFTH TWO DIGIT

37

From the last diagram
State Metrics

$\Gamma_a = 2$

$\Gamma_b = 2$

$\Gamma_c = 2$

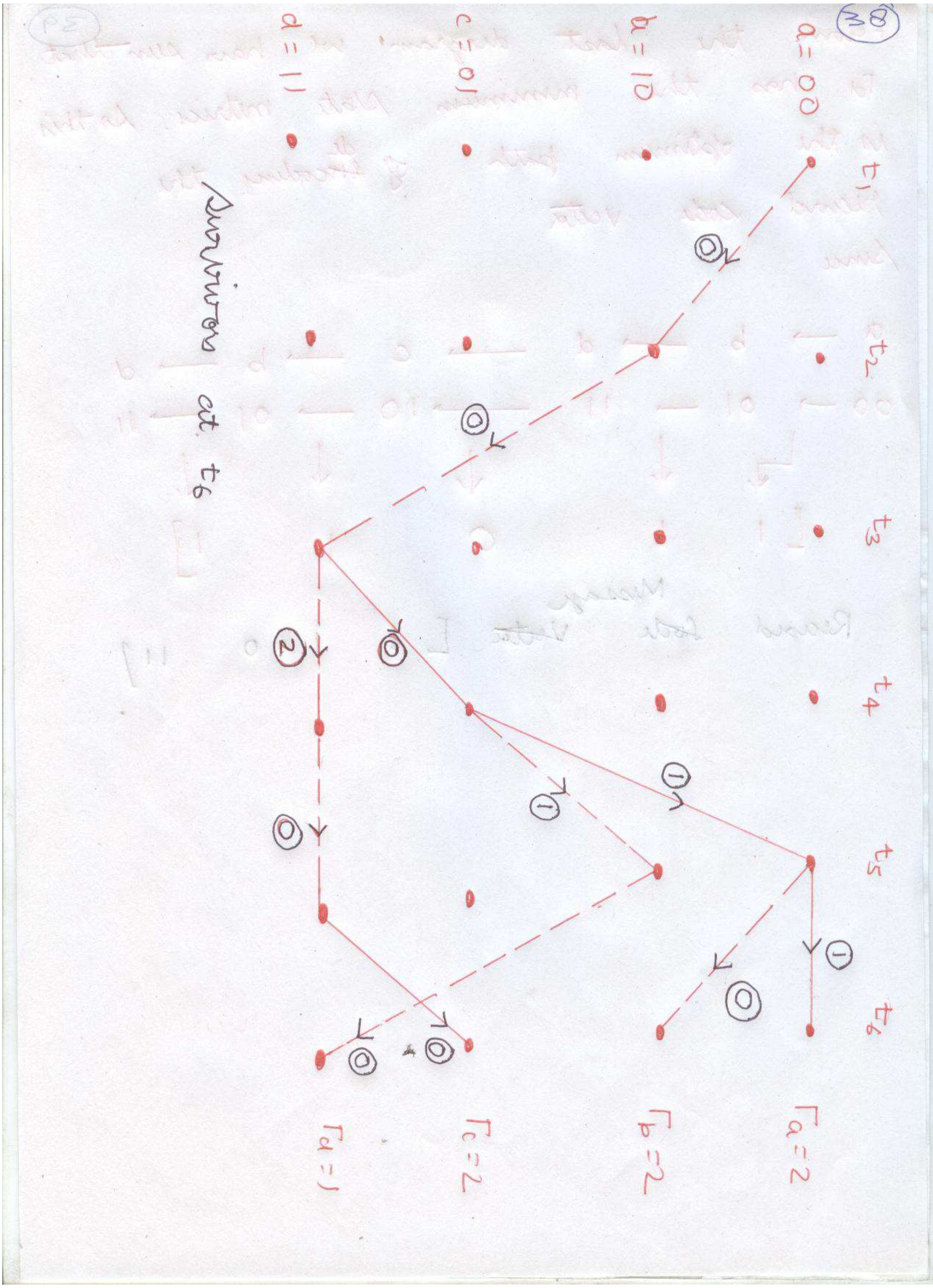
$\Gamma_d = 1$

$\Gamma_e = 4$

$\Gamma_f = 4$

$\Gamma_g = 2$

$\Gamma_h = 4$


a = 00
b = 10
c = 01
d = 11
t1
t2
t3
t4
t5
t6
Survivors at t6
Γa = 2
Γb = 2
Γc = 2
Γd = 1

(39)

From the last diagram we have seen that Td has the minimum state metrics. So this is the optimum path of decoding the received code vector

Since

a → b → d → c → b → d

00 → 01 → 11 → 10 → 01 → 11

[1 1 0 1 1]

Received Message Code Vector [1 1 0 11]

Solutions of Mathematical Problem of Assignment-3

DATE - (1)

1- From the problem it is clear that there are four message bit and four parity check bits. The complete code vector look like

$$[X] = [m_1\ m_2\ m_3\ m_4\ p_1\ p_2\ p_3\ p_4]$$

We know that

$$[C] = [M][P]$$

$$[p_1\ p_2\ p_3\ p_4] = [m_1\ m_2\ m_3\ m_4]\ [P]$$

$$= [m_1\ m_2\ m_3\ m_4] \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}$$

$$P_1 = [(m_1 P_{11}) \oplus (m_2 P_{21}) \oplus (m_3 P_{31}) \oplus (m_4 P_{41})]$$

$$P_2 = [(m_1 P_{12}) \oplus (m_2 P_{22}) \oplus (m_3 P_{32}) \oplus (m_4 P_{42})]$$

$$P_3 = [(m_1 P_{13}) \oplus (m_2 P_{32}) \oplus (m_3 P_{33}) \oplus (m_4 P_{43})]$$

$$P_4 = [(m_1 P_{14}) \oplus (m_2 P_{24}) \oplus (m_3 P_{34}) \oplus (m_4 P_{44})]$$

Comparing with the data from the problem (2)

$P_{11} = 1$ $P_{12} = 1$ $P_{13} = 1$ $P_{14} = 0$

$P_{21} = 1$ $P_{22} = 0$ $P_{23} = 1$ $P_{24} = 1$

$P_{31} = 0$ $P_{32} = 1$ $P_{33} = 1$ $P_{34} = 1$

$P_{41} = 1$ $P_{42} = 1$ $P_{43} = 0$ $P_{44} = 1$

Now

$$[P] = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Generator Matrix

$$[G] = [P : I_k]$$

$$[G] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Parity Check Matrix $[H] = [P^T : I_{n-k}]$

$$[H] = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculation of all code vectors (3)

	m_1	m_2	m_3	m_4	P_1	P_2	P_3	P_4	w(x)
0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	0	1	4
2	0	0	1	0	0	1	1	1	4
3	0	0	1	1	1	0	1	1	5
4	0	1	0	0	1	0	1	1	4
5	0	1	0	1	0	1	1	1	4
6	0	1	1	0	1	1	0	0	4
7	0	1	1	1	0	0	0	1	4
8	1	0	0	0	1	1	1	0	4
9	1	0	0	1	0	0	1	1	4
10	1	0	1	0	1	0	0	1	4
11	1	0	1	1	0	1	0	0	4
12	1	1	0	0	0	1	0	1	4
13	1	1	0	1	1	0	0	0	4
14	1	1	1	0	0	0	1	0	4
15	1	1	1	1	1	1	1	1	8

$d_{min} = w(x)|$ Non Zero Code Vector

$d_{min} = 4$

$d_{min} \geq (2t+1)$

$t \geq 1.5$

One Error can be correct with the help of this code

$$[H^T] = \begin{bmatrix} P \\ I_{(n-k)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S = [Y][H^T] = [1\,0\,1\,0\,1\,0\,1\,0][H^T]$$

$$= [\{1 \oplus 0 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0\}; \{1 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0\}; \{1 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 1 \oplus 0\}; \{0 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0\}]$$

$$= [0 \; 0 \; 1 \; 1]$$

Not a valid code vector

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