Solution of Mathematical Problems of Minor -2 (01)

(01) From the problem

$$E = 10(\bar{a}_z + \bar{a}_y)\cos[(2\pi\times10^7)t - \beta x]$$

$$\mu = 50\mu_0 \qquad \epsilon = 2\epsilon_0$$

$$\sigma = 0$$

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{[(50\mu_0)(2\epsilon_0)]}}$$

$$= \frac{1}{10\sqrt{\mu_0\epsilon_0}} = \frac{c}{10}$$

$$= 3\times10^7 \text{ mt/sec}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1+\left(\frac{\sigma}{w\epsilon}\right)^2\right]^{1/4}} = \frac{\sqrt{\mu_0/\epsilon_0}\,\sqrt{25}}{[1+0]^{1/4}}$$

$$= 5\times120\pi = 600\pi$$

$$\angle\theta_\eta = \frac{1}{2}\tan^{-1}\left|\frac{\sigma}{w\epsilon}\right| = \frac{1}{2}\tan^{-1}(0)$$

$$= 0$$

$$\eta = |\eta|\,\epsilon^{\angle\theta_\eta} = 600\pi\ \Omega$$

$$\beta = \frac{2\pi}{\lambda} = \frac{w}{v_p} = \frac{2\pi\times10^7}{3\times10^7}$$

$$\boxed{\beta = \frac{2\pi}{3}}$$

02

$$a_K = a_x$$
$$a_E = (a_z + a_y)$$
$$a_H = a_K \times a_E$$
$$= a_x \times (a_z + a_y)$$
$$= [(a_x \times a_z) + (a_x \times a_y)]$$
$$= [- a_y + a_z]$$

$$H = \frac{E}{\eta}$$

$$\bar{H} = \frac{1}{60\pi} \left[\cos [8\omega t - \beta x] \ (-\bar{a}_y + \bar{a}_z) \right]$$

where $\omega = 2\pi \times 10^7$

$$\beta = \frac{2\pi}{3}$$

03 We know that input impedance is calculated by

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

For the first transmission line

$$l = \lambda/4$$

$$\tan\beta l_1 = \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{4}\right) = \tan\left(\frac{\pi}{2}\right)$$
$$= \infty$$

$$Z_{in_1} = Z_0 \left[\frac{\frac{Z_L}{\tan\beta l_1} + jZ_0}{\frac{Z_0}{\tan\beta l_1} + jZ_L} \right]$$

03

$$Z_{in_1} = \frac{Z_0^2}{Z_L} = \frac{50 \times 50}{200}$$

$$= \frac{25}{2}\,\Omega$$

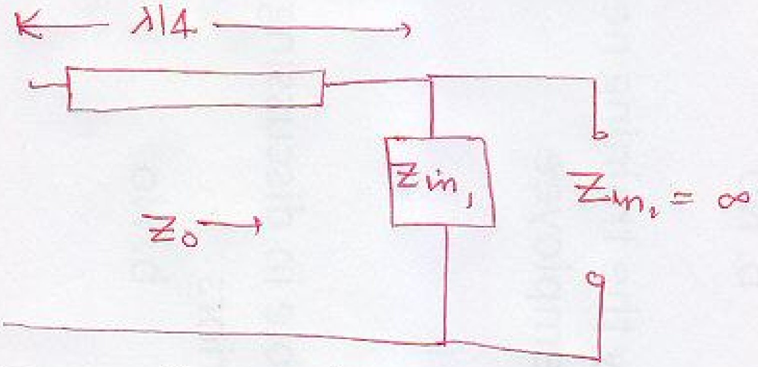
For the second transmission line which has zero load impedance then

$$Z_{in_2} = j Z_0 \tan\beta l_2$$

$$Z_{in_2} = \infty$$

For the second line input impedance is ∞

Redrawing the transmission line.



From the above figure

$$Z_{L_3} = Z_{in_1} = \left(\frac{25}{2}\right)\Omega$$

$$l_3 = \lambda/4$$

$$\tan\beta l_3 = \infty$$

Now input impedance is given by

$$Z_{in_3} = Z_0 \left[\frac{\dfrac{Z_L}{\tan\beta l_3} + j Z_0}{\dfrac{Z_0}{\tan\beta l_3} + j Z_L} \right]$$

$$= \frac{Z_0^2}{Z_L} = \frac{50 \times 50}{25} \times \Omega \quad (04)$$

$$= 200\Omega$$

$$Z_{in} = Z_{in_3} = 200\Omega$$

(04)

$$f = 400 \text{ MHz}$$
$$= [400 \times 10^6] \text{ Hz}$$

$$R + j\omega L = [3.5 + j(2\pi f L)]$$
$$= [3.5 + j(2 \times 3.14 \times 4 \times 10^8 \times 2 \times 10^{-6})]$$
$$= [3.5 + j(1600\pi)]$$

$$G + j\omega C = j(2\pi f)C$$
$$= j(2\pi \times 4 \times 10^8 \times 120 \times 10^{-9})$$
$$= j(96\pi)$$

Characteristic Impedance

$$Z_0 = \sqrt{\left[\frac{R + j\omega L}{G + j\omega C}\right]}$$
$$= \sqrt{\left[\frac{3.5 + j(1600\pi)}{j(96\pi)}\right]}$$
$$= \sqrt{[16.67 - 0.01j]}$$
$$= [16.67 \angle -0.04]^{1/2}$$

$$= [4.08 \angle -0.02]\,\Omega$$

05

Propagation constant calculated by

$$\gamma = \alpha + j\beta = \sqrt{[(R+j\omega L)(G+j\omega C)]}$$

$$= \sqrt{[(3.5 + j1600\pi)][j\,96\pi]}$$

$$= \sqrt{[-1514434.56 + j\,1055.04]}$$

$$= \sqrt{[1514434.928 \angle -179.98]}$$

$$= 12306.63 \angle -89.98$$

$$= [4.296 - 12306.63j]$$

$$\alpha = 4.296$$

$$\beta = -12306.63$$ (-ve represent direction only)

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$$

$$= \left[\frac{2 \times 3.14 \times 4 \times 10^8}{12306.63}\right]$$

$$= 204.12 \times 10^3 \text{ mt/sec}$$

(06) Input impedance is given by (06)

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

$$\tan\beta l = \tan\left[\frac{2\pi}{\lambda} \times \frac{5\lambda}{8}\right] = \tan\left[\frac{5\pi}{4}\right]$$

$$= \tan\left[\pi + \frac{\pi}{4}\right] = \tan\left[\frac{\pi}{4}\right] = 1$$

(I) $Z_L = 75\Omega$

$$Z_{in} = 75\left[\frac{j45 + j75}{75 + j(j45)}\right] = 75\left[\frac{j120}{30}\right]$$

$$= 75 \times 4j = [300j]\Omega$$

(II) $Z_L = (25 - j65)$

$$Z_{in} = 75\left[\frac{(25 - j65) + j75}{75 + j(25 - j65)}\right]$$

$$= 75\left[\frac{25 + j10}{140 + j25}\right]$$

$$= [13.9 + 2.87j]\Omega$$