Solution of Assignment - 3

(01)

PE 8.4 -

Since the medium is free space then

$\mu_r = 1 \qquad \epsilon_r = 1$

so $\mu = \mu_0 \qquad \epsilon = \epsilon_0$

$$J_d = \frac{\partial \bar{D}}{\partial t} = \epsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$= -\epsilon_0 \times 20 \times w \times \sin(wt - 50x)\, \bar{a}_y$$

$$J_d = -20 w \epsilon_0 \sin(wt - 50x)\, \bar{a}_y \text{ A/mt}^2$$

Wave propagation direction $a_K = a_x$

Electric field direction $a_E = \bar{a}_y$

Magnetic field direction $a_H = a_K \times a_E$

$= \bar{a}_x \times \bar{a}_y = \bar{a}_z$

Since the direction of magnetic field is +Z axis, then only H_z component is present in the magnetic field.

$$\nabla \times \bar{H} = J_d + J_c$$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & H_z \end{vmatrix} = J_d$$

since $J_c = 0$

$$\Rightarrow \bar{a}_x\left[\frac{\partial H_z}{\partial y} - 0\right] - \bar{a}_y\left[\frac{\partial H_z}{\partial x} - 0\right] + \bar{a}_z[0-0]$$ (02)

$$= -20 w \epsilon_0 \sin(wt - 50x)\,\bar{a}_y$$

comparing both side

$$-\frac{\partial H_z}{\partial x} = -20 w \epsilon_0 \sin(wt - 50x)$$

$$\Rightarrow H_z = \frac{20 w \epsilon_0}{1}\int \sin(wt - 50x)\,dx$$

$$\Rightarrow H_z = 0.4 w \epsilon_0 \cos(wt - 50x)$$

$$\bar{H} = [0.4 w \epsilon_0 \cos(wt - 50x)]\,\bar{a}_z$$

Now

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t} = -\mu_0 \frac{\partial H}{\partial t}$$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix} = -\mu_0 \frac{\partial H}{\partial t}$$

$$\Rightarrow \bar{a}_x\left[0 - \frac{\partial E_y}{\partial z}\right] - \bar{a}_y[0-0] + \bar{a}_z\left[\frac{\partial E_y}{\partial x} - 0\right]$$

$$= -0.4 w^2 \epsilon_0 \mu_0 \sin(wt - 50x)\ \bar{a}_z$$

comparing the both end

(03)

$$\frac{\partial E_y}{\partial x} = -0.4\,\mu_0 \epsilon_0 W^2 \sin(Wt - 50x)$$

$$\Rightarrow -1000 \sin(Wt - 50x) = -0.4 \frac{W^2}{c^2} \sin(Wt - 50x)$$

$$\Rightarrow 1000 = 0.4 \frac{W^2}{c^2}$$

$$\Rightarrow 2500 \times c^2 = W^2$$

$$\Rightarrow 50c = W$$

$$W = 1.5 \times 10^{10} \text{ Rad/Sec}$$

(9.9)

From the problem

$$\alpha = 0.1 \text{ Np/mt}$$

$$\eta = 250 \angle 35.26$$

$$|\eta| = 250$$

$$\angle\theta_\eta = 35.26$$

Loss Angle $\theta = 2\theta_\eta = 70.52'$

Loss Tangent $\tan 2\theta_\eta = \tan 70.52$

$$= 2.827$$

$$\tan 2\theta_\eta = \left(\frac{\sigma}{W\epsilon}\right) = 2.827$$

We know that

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+\left(\frac{\sigma}{W\epsilon}\right)^2} - 1}{\sqrt{1+\left(\frac{\sigma}{W\epsilon}\right)^2} + 1}\right]^{0.5}$$

(8) $\frac{\alpha}{\beta} = \left[\frac{\sqrt{9}-1}{\sqrt{9}+1}\right]^{0.5} = \left[\frac{3-1}{3+1}\right]^{0.5}$ (04)

$$\frac{\alpha}{\beta} = \frac{1}{\sqrt{2}}$$

$$\beta = [\alpha\sqrt{2}] = [0.1\times\sqrt{2}] = 0.14$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.1\times\sqrt{2}} = 44.43\,mt$$

(9.10)

Magnetic field is given as

$$H = 0.2e^{-y}\cos[2\pi\times10^{8}t - 5y]\,\bar{a}_z$$

$\alpha = 1$ $\beta = 5$

$\mu_r = 1$

$a_K = \bar{a}_y$ $a_H = \bar{a}_z$

$a_E = \bar{a}_H \times \bar{a}_K$

$= \bar{a}_z \times \bar{a}_y = -\bar{a}_x$

$w = 2\pi\times10^{8}$

We know that

$$\frac{\alpha}{\beta} = \left[\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right]^{0.5} \quad x = \left(\frac{\sigma}{w\epsilon}\right)$$

$$\Rightarrow \frac{1}{25} = \frac{m-1}{m+1} \quad m = \sqrt{(1+x^2)}$$

$$\Rightarrow (m+1) = 25m - 25$$

(05)

(10)

$$m = \frac{13}{12}$$

$$\Rightarrow \quad 1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2 = \frac{169}{144}$$

$$\Rightarrow \quad \left(\frac{\sigma}{\omega\epsilon}\right)^2 = \frac{25}{144}$$

$$\Rightarrow \quad \frac{\sigma}{\omega\epsilon} = \frac{5}{12}$$

We know that

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\Rightarrow \quad \alpha = \omega\sqrt{\frac{\mu_r\epsilon_r}{2c^2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\Rightarrow \quad \frac{\alpha c}{\omega} = \sqrt{\frac{\epsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\Rightarrow \quad \frac{\alpha c}{\omega} = \sqrt{\frac{\epsilon_r}{2}\left[\frac{13}{12} - 1\right]}$$

$$\Rightarrow \quad \left(\frac{\alpha c}{\omega}\right)^2 = \frac{\epsilon_r}{12}$$

$$\Rightarrow \quad \epsilon_r = \left[\pm 24\left(\frac{\alpha c}{\omega}\right)^2\right]$$

$$\Rightarrow \quad \epsilon_r = +24 \times \left(\frac{1 \times 3\times10^8}{2\pi\times10^8}\right)^2$$

$$\Rightarrow \quad \epsilon_r = 5.471$$

(06)

$$\frac{\sigma}{w\epsilon} = \frac{5}{12}$$

$$\sigma = \frac{5}{12} w\epsilon = \frac{5}{12} w \epsilon_0 \epsilon_r$$

$$= \frac{5}{12} \times 2\pi \times 10^8 \times 5.471 \times \frac{10^{-9}}{36\pi}$$

$$= 0.0127 \ S/mt^2$$

$$\bar{E} = [\eta \times H] \bar{a}_E$$

$$|\eta| = \left[\frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{w\epsilon}\right)^2\right]^{0.25}} \right]$$

$$= \left[\frac{\frac{120\pi}{\sqrt{5.471}}}{\sqrt{(13/12)}} \right] = 154.85$$

$$\tan 2\theta_\eta = \left|\frac{\sigma}{w\epsilon}\right| = \frac{5}{12}$$

$$\theta_\eta = 11.31$$

$$\bar{E} = [154.85 \angle 11.31] [0.2 e^{-y} \cos(2\pi \times 10^8 t - 5y)] [-\bar{a_x}]$$

$$\Rightarrow \bar{E} = -30.97 e^{-y} \cos[2\pi \times 10^8 t - 5y] \bar{a_x}$$

Q.22 Wave propagate in free space region 07

$$\rho_v = 0$$

Field equations are given as

$$E = E_0 e^{j(K.r - wt)}$$

$$H = H_0 e^{j(K.r - wt)}$$

$$\bar{K} = [K_x \bar{a}_x + K_y \bar{a}_y + K_z \bar{a}_z]$$

$$\bar{r} = [x \bar{a}_x + y \bar{a}_y + z \bar{a}_z]$$

$$\bar{K}.\bar{r} = [x K_x + y K_y + z K_z]$$

Now

$$E = E_0 e^{j(K_x x + K_y y + K_z z - wt)}$$

First Maxwell equation is given as

$$\nabla . \bar{D} = \rho_v$$

$$\Rightarrow \nabla . (\epsilon \bar{E}) = 0$$

$$\Rightarrow \epsilon_0 (\nabla . \bar{E}) = 0$$

$$\Rightarrow \nabla . \bar{E} = 0$$

$$\Rightarrow \left[\bar{a}_x \frac{\partial}{\partial x} + \bar{a}_y \frac{\partial}{\partial y} + \bar{a}_z \frac{\partial}{\partial z}\right] . [\bar{E}] = 0$$

$$\Rightarrow E_0 \left[\bar{a}_x \frac{\partial}{\partial x} + \bar{a}_y \frac{\partial}{\partial y} + \bar{a}_z \frac{\partial}{\partial z}\right] . \left[e^{j(K_x x + K_y y + K_z z - wt)}\right]$$

$$E_0 = 0$$

$$\Rightarrow j(K_x \bar{a}_x + K_y \bar{a}_y + K_z \bar{a}_z)\left[e^{j(K_x x + K_y y + K_z z - wt)}\right] E_0 = 0$$ (08)

$$\Rightarrow j(K_x \bar{a}_x + K_y \bar{a}_y + K_z \bar{a}_z).\bar{E} = 0$$

$$\Rightarrow j\bar{K}.\bar{E} = 0$$

$$\Rightarrow \boxed{\bar{K}.\bar{E} = 0}$$

In the similar way

$$\nabla.H = jK.H = 0$$

$$K.H = 0$$

Third Maxwell equation

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t} = -\mu\frac{\partial H}{\partial t}$$

$$\Rightarrow \nabla \times \bar{E} = -\mu\frac{\partial}{\partial t}\left[H_0 e^{j(K.r - wt)}\right]$$

$$\Rightarrow \nabla \times \bar{E} = jw\mu H_0 e^{j(Kr - wt)}$$

$$\Rightarrow \nabla \times \bar{E} = jw\mu\bar{H}$$

$j \rightarrow$ Taken as phase shift
$j = e^{j(\pi/2)}$

$$\boxed{\nabla \times \bar{E} = w\mu\bar{H}}$$

In the similar way

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}.$$

09

$$\Rightarrow \nabla \times \bar{H} = -j\epsilon\omega\bar{E}$$

$$\Rightarrow \boxed{\nabla \times \bar{H} = -\epsilon\omega\bar{E}}$$

9.23 Wave equation represented as

$$\bar{H} = 25\cos(40,000\,t + \beta z)\,\bar{a}_y \text{ A/mt}$$

$25 \rightarrow H_0$, $40,000 \rightarrow \omega$, $\bar{a}_y \rightarrow a_H$

$$\alpha = 0$$

$$a_K = -\bar{a}_z$$

$$a_E = a_H \times a_K = \bar{a}_y \times (-\bar{a}_z)$$

$$= -\bar{a}_x$$

SHRISH BAJPAI
Assistant Professor
of Electronics & Communication Engineering
Engineering, Integral University, Lucknow

Since medium is free space then

$$\mu_r = 1 \qquad \epsilon_r = 1$$

$$v_p = c \quad , \quad \eta = 120\pi$$

$$\beta = \frac{\omega}{v_p} = \frac{40,000}{c} = \left[\frac{40,000}{3\times10^8}\right]$$

$$\beta = 1.33\times10^{-4} \text{ RAD/SEC}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\frac{4}{3}\times10^{-4}}$$

$$= 4.712\times10^4 \text{ mt/sec}$$

We know that

$$\bar{E} = [\eta \bar{H}]\,\bar{a}_E.$$

$= 120\pi \times 25 \cos(40000t + \beta z)(-\bar{a}_x)$ KV/mt (10)

$= -9.425 \times 10^3 \cos(40000t + \beta z)\, \bar{a}_x$ KV/mt

9.38 From the problem it is clear that

$$\delta = \sqrt{\frac{2}{w\mu\sigma}}$$

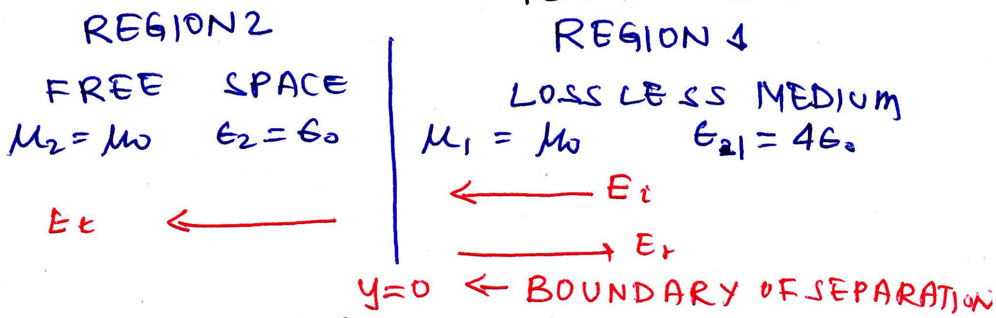
$$w = \left[\frac{2}{\delta^2 \mu \sigma}\right] = \left[\frac{2}{4\pi\times10^{-7} \times 5.8\times10^{7} \times (10^{-4})^2}\right]$$

$$= 2.744 \times 10^6 \text{ RAD/SEC}$$

$$f = \left[\frac{w}{2\pi}\right] = \left[\frac{2.744\times10^6}{2\pi}\right]$$

$$= 436.7 \text{ KHz}$$

9.53



$$E_i = 5 \cos(10^8 t + \beta y)\, \bar{a}_z$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = 120\pi\sqrt{\frac{1}{4}} = 60\pi$$

$$\eta_2 = 120\pi \text{ [FREE SPACE]}$$

$$\Gamma = \left[\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right] = \frac{120\pi - 60\pi}{120\pi + 60\pi} = \frac{60\pi}{180\pi} = \frac{1}{3}$$

$$\tau = \left[\frac{2\eta_2}{\eta_2 + \eta_1}\right] = \left[\frac{2\times120\pi}{120\pi + 60\pi}\right] = \frac{4}{3}$$

(11)

$$E_{io} = 10$$

$$E_{ro} = \Gamma E_{io} = 10/3$$

$$E_{to} = \tau E_{io} = 40/3$$

$$a_{E_i} = \bar{a}_z \qquad a_{E_t} = a_{E_i} = \bar{a}_z$$

$$a_{E_r} = -a_{E_i} = -\bar{a}_z$$

$$P_1 = [P_i + P_r]$$

$$= \frac{E_i^2}{2\eta_1}\bar{a}_z + \frac{E_r^2}{2\eta_1}(-\bar{a}_z)$$

$$= \frac{100}{120\pi}\bar{a}_z - \frac{100}{9\times 60\pi}\bar{a}_z$$

$$= 0.235\,\bar{a}_z \text{ W/mt}^2$$

$$P_2 = P_t$$

$$= \frac{E_t^2}{2\eta_2}\bar{a}_z$$

$$= \frac{1600}{9\times 2\times 120\pi}\bar{a}_z$$

$$= 0.2358\,\bar{a}_z \text{ W/mt}^2$$

(9.60) Incident wave is given as

$$E_i = [8\bar{a}_x + 6\bar{a}_y + 5\bar{a}_z]\sin(wt + 3x - 4y) \text{ v/mt}$$

at $y < 0$ and Medium is free space medium

$$\beta_1 = \sqrt{3^2 + 4^2} = 5$$

$$\beta_1 = \frac{w}{v_p} = \frac{w}{c}$$

$$w = \beta_1 c = 15\times 10^8 \text{ RAD/SEC}$$

Let reflected wave given as (12)

$$E_r = [E_{rx}\bar{a}_x + E_{ry}\bar{a}_y + E_{rz}\bar{a}_z]\sin(wt + 3x + 4y)$$

Because wave reflected with respect to the Y axis

Reflected wave travel in the same medium of incident wave (free space medium)

$$\nabla\cdot\bar{D}_r = \rho_v$$

$$\Rightarrow \epsilon[\nabla\cdot E_r] = \rho_v$$

$$\Rightarrow \nabla\cdot\bar{E}_r = 0$$

$$\Rightarrow \left[\bar{a}_x\frac{\partial}{\partial x} + \bar{a}_y\frac{\partial}{\partial y} + \bar{a}_z\frac{\partial}{\partial z}\right]\cdot[E_{0x}\bar{a}_x + E_{0y}\bar{a}_y + E_{0z}\bar{a}_z]\sin(wt + 3x + 4y) = 0$$

$$\Rightarrow 4E_{0y} + 3E_{0x} = 0$$

$t = 0$

$$E_{1tan} = E_{2tan}$$

$$E_{1tan} = 0$$

} Boundary condition of electrostatic fields

$$8\bar{a}_x + 5\bar{a}_z + E_{0x}\bar{a}_x + E_{0z}\bar{a}_z = 0$$

$$E_{0x} = -8$$

$$E_{0z} = -5$$

$$E_{0y} = 6$$

Reflected wave given as

$$E_r = [-8\bar{a}_x + 6\bar{a}_y - 5\bar{a}_z]\sin(wt + 3x + 4y)\ V/mt^2$$