Mathmatical Solution of the problems of Assignment – 4

01

10.37 From the problem

$$Z_0 = 75\,\Omega$$

$$f = 150 \times 10^6 \text{ Hz}$$

$$\alpha = 0.06 \text{ Np/mt}$$

$$v_p = 2.8 \times 10^8 \text{ mt/sec}$$

Since the line is distortionless line then

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$$

$$\alpha = \sqrt{RG}$$

$$v_p = \frac{1}{\sqrt{LC}}$$

$$\alpha Z_0 = R$$

$$\Rightarrow \quad R = 0.06 \times 75 = 4.5\,\Omega/\text{mt}$$

$$\sqrt{RG} = \alpha$$

$$\Rightarrow \quad G = \frac{\alpha^2}{R} = \frac{36 \times 10^{-4}}{4.5}$$

$$= 8 \times 10^{-4} \text{ S/mt}$$

Now,

$$v_p Z_0 = \frac{1}{C} = 75 \times 2.8 \times 10^8$$

$$C = \frac{10^{-8}}{75 \times 2.8}$$

$$= 4.76 \times 10^{-11} \text{ F/mt}$$

$$L = \frac{Z_0}{v_p} = \frac{75}{2.8 \times 10^8} = 2.678 \times 10^{-7} \text{ H/mt}$$ (02)

(10.20) From the problem, it is given

$Z_0 = 40\Omega$

When the line is behave like as short circuit $Z_{sc} =$ then,

$$Z_{sc} = jZ_0 \tan \beta l$$

$$\Rightarrow j73 = j40 \tan \beta l$$

$$\Rightarrow \tan \beta l = \frac{73}{40} = 1.738$$

$$\Rightarrow \tan \beta l = \tan 60$$

$$\Rightarrow \beta l = \frac{\pi}{3}$$

$$\Rightarrow \frac{2\pi}{\lambda} \times l = \frac{\pi}{3}$$

$$\Rightarrow l = \lambda/6$$

When the line is behave like as open circuit then,

$$Z_{oc} = -jZ_0 \cot \beta l$$

$$\Rightarrow j73 = -j42 \cot \beta l$$

$$\Rightarrow \cot \beta l = -1.738$$

$$\Rightarrow \cot \beta l = \cancel{\cot\left(\frac{2\pi}{3}\right)} \cot\left(\frac{5\pi}{6}\right)$$

$$\Rightarrow \beta l = \frac{5\pi}{6}$$

$$\Rightarrow \frac{2\pi}{\lambda} \times l = \frac{5\pi}{6}$$

$$\Rightarrow l = \frac{5\lambda}{12}$$

03

10.30 Three transmission line is connected as shown in the figure. The input impedence of the last (third) waveguide is the load impedence of the second waveguide.

Now for the third waveguide

$$Z_{0_3} = 75\,\Omega$$

$$Z_{L_3} = (60 - 40j)\,\Omega$$

$$l_3 = (3\lambda/4)$$

For any waveguide, the input impedence is calculated by

$$Z_{in_{\cancel{3}}} = Z_{0_{\cancel{3}}}\left[\frac{Z_{L_{\cancel{3}}} + jZ_{0_{\cancel{3}}}\tan\beta l_{\cancel{3}}}{Z_{0_{\cancel{3}}} + jZ_{L_{\cancel{3}}}\tan\beta l_{\cancel{3}}}\right]$$

For third waveguide

$$\tan\beta l_3 = \tan\left(\frac{2\pi}{\lambda}\times\frac{3\lambda}{4}\right)$$

$$= \tan\left(\frac{3\pi}{2}\right) = \infty$$

So,

$$Z_{in_3} = Z_{0_3}\left[\frac{Z_{L_3} + jZ_{0_3}\tan\beta l_3}{Z_{0_3} + jZ_{L_3}\tan\beta l_3}\right]$$

$$= Z_{0_3}\left[\frac{\frac{Z_{L_3}}{\tan\beta l_3} + jZ_{0_3}}{\frac{Z_{0_3}}{\tan\beta l_3} + jZ_{L_3}}\right]$$

04

$$\Rightarrow \quad Z_{in_3} = Z_{0_3} \times \frac{Z_{0_3}}{Z_{L_3}}$$

$$\Rightarrow \quad Z_{in_3} = \frac{Z_{0_3}^2}{Z_{L_3}}$$

For the 2nd waveguide

$$Z_{L_2} = Z_{in_3}$$

$$l_2 = \lambda/2$$

$$\tan \beta l_2 = \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2}\right) = \tan \pi = 0$$

$$Z_{in_2} = Z_{0_2}\left[\frac{Z_{L_2} + j Z_{0_2} \tan \beta l_2}{Z_{0_2} + j Z_{L_2} \tan \beta l_2}\right]$$

$$= Z_{0_2} \times \frac{Z_{L_2}}{Z_{0_2}} = Z_{L_2}$$

So $\quad Z_{in_2} = Z_{L_2} = \left[\frac{Z_{0_3}^2}{Z_{L_3}}\right]$

For the first waveguide

$$Z_{L_1} = Z_{in_2} = \frac{Z_{0_3}^2}{Z_{L_2}}$$

$$l_1 = \lambda/4$$

Now $\quad \tan \beta l_1 = \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{4}\right)$

$$= \tan\left(\frac{\pi}{2}\right) = \infty$$

$$Z_{in_1} = Z_{0_1}\left[\frac{Z_{L_1} + j Z_0 \tan \beta l_1}{Z_{0_2} + j Z_{L_1} \tan \beta l_1}\right]$$

$$= Z_{o_1}\left[\frac{\dfrac{Z_{L_1}}{\tan\beta l_1} + j Z_{o_1}}{\dfrac{Z_{o_1}}{\tan\beta l_1} + j Z_{L_1}}\right]$$ 05

$$= Z_{o_1}\left[\frac{Z_{o_1}}{Z_{L_1}}\right] = \frac{Z_{o_1}^2}{Z_{L_1}}$$

$$= \frac{Z_{o_1}^2}{Z_{o_2}^2} \times Z_{L_3}$$

$$= \frac{50 \times 50}{75 \times 75} \times (60 - 40j)$$

$$= \frac{4}{9}(60 - 40j)\ \Omega$$

10.32 From the problem

$$Z_o = 75\Omega$$

$$Z_L = [120 + j80]\Omega$$

Reflection coefficient

$$\Gamma = \left[\frac{Z_L - Z_o}{Z_L + Z_o}\right] = \left[\frac{120 + j80 - 75}{120 + j80 + 75}\right]$$

$$= 0.435 \angle 38.34$$

$$VSWR = \left[\frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}\right] = \left[\frac{1 + 0.435}{1 - 0.435}\right]$$

$$= 2.54$$

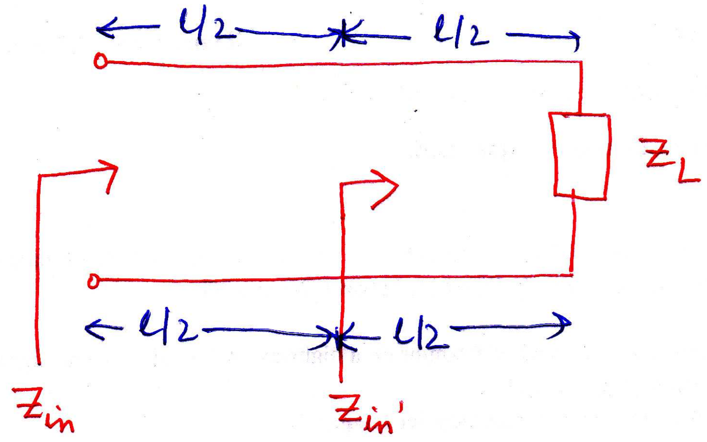
The load is resistive when

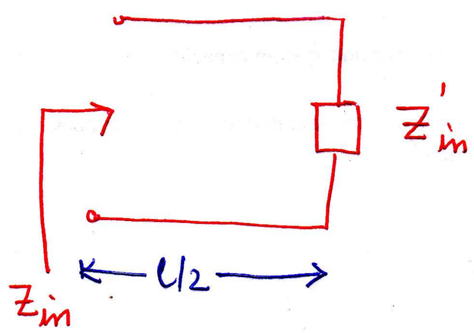
$$720^\circ \longrightarrow \lambda$$

$$38^\circ \longrightarrow \frac{38\lambda}{720} = 0.053\lambda$$

10.35 From the problem (06)

$$Z_0 = 50\,\Omega$$
$$l = 4.2\,mt$$
$$f = 300\ MHz = 3\times10^8\ Hz$$
$$v_p = 0.8c = 2.4\times10^8\ mt$$





From the figure, it is clear that input impedence of a waveguide work as the load impedence of the another series connected waveguide.

$$Z_L = Z_{in}' = (80-60j)\,\Omega$$

$$\tan(\beta l) = \tan\left(\frac{2\pi}{\lambda}\times 2.1\right)$$

07

$$= \tan\left(\frac{2\pi f}{c} \times 2.1\right)$$

$$= \tan\left[\frac{2\pi \times 3\times 10^8}{2.4\times 10^8} \times 2.1\right]$$

$$= \tan\left[\frac{4.2\pi}{0.8}\right] = \tan\left(\frac{21\pi}{4}\right)$$

$$= \tan\left(5\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

Input impedence of the waveguide

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$= 50\left[\frac{80 - 60j + 50j}{50 + j80 + 60}\right]$$

$$= [21.6 - j20.2]\,\Omega$$

$$\Gamma = \left[\frac{Z_L - Z_0}{Z_L + Z_0}\right] = \left[\frac{80 - 60j - 50}{80 - 60j + 50}\right]$$

$$= 0.4685 \angle -38.66$$

$$\theta_L = -38.66$$

$$\theta_L' = \theta_L + 2\times(\pi/4)$$

$$= 51.34^\circ$$

Reflection coefficient 08

$$\Gamma = 0.485 \angle 51.34$$

10.39

$$VSWR = \left[\frac{V_{max}}{V_{min}}\right] = \frac{0.8}{0.5} = 1.6$$

$$\frac{\lambda}{2} = [19 - 9.5] = 9.5 \text{ c.m.}$$

$$\lambda = 19 \text{ c.m.}$$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{19 \times 10^{-2}} = 1.58 \text{ GHz}$$

$$l = 19 - 14 = 5 \text{ c.m.}$$

$$= \frac{5\lambda}{19} = 189.4$$

$$Z_L = Z_0 z_L = 50 [1.59 + j0.14]$$

$$= [79.5 + j7] \Omega$$

$$|\Gamma| = \left[\frac{VSWR - 1}{VSWR + 1}\right] = \frac{1.6 - 1}{1.6 + 1}$$

$$= \frac{0.6}{2.6} = 0.231$$

10.45

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - 50}{30 + 50}$$

$$= -\frac{20}{80} = -\frac{1}{4}$$

09

$$\Gamma_g = \left[\frac{Z_g - Z_0}{Z_g + Z_0}\right] = \left[\frac{75-50}{75+50}\right]$$

$$= \frac{25}{125} = \frac{1}{5}$$

$$t_1 = \frac{l}{c} = \frac{600}{3\times10^8} = 2\mu s$$

$$V_0 = \left[\frac{Z_0}{Z_0+Z_g}\right] V_g = \frac{50\times30}{125} = \frac{10\times30}{\cancel{50}25}$$

$$= 12\,\text{Volt}$$

$$V_\infty = \left[\frac{Z_L}{Z_L+Z_0}\right] \times V_g = \frac{30\times30}{75+30}$$

$$= 8.571\,\text{Volt}$$

$$I_0 = \frac{V_0}{Z_0} = \frac{12}{50} = 240\,mA$$

$$I_\infty = \frac{V_\infty}{Z_L} = \frac{30}{105} = 285.7\,mA$$

10.48

$$V_\infty = \frac{Z_L}{Z_L+Z_g} \times V_g$$

$$= \left[\frac{100\times200}{100+50}\right] = 133.33\,V$$

$$I_\infty = \frac{V_\infty}{Z_L} = 133.33$$

10.57

10

$$L = \left[\frac{Z_0}{v_p}\right] = \frac{Z_0\sqrt{\epsilon}}{c}$$

$$= \frac{60\sqrt{5}}{3\times10^{8}} = 1\ \mu H/mt$$

$$C = \frac{1}{Z_0 v_p} = \frac{\sqrt{\epsilon}}{Z_0\, c}$$

$$= \frac{\sqrt{5}}{60\times3\times10^{8}} = 0.1242\ nF/mt$$